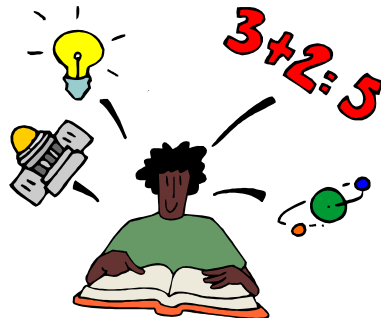


Written Calculation Policy

Key Stage 1 and 2



Kibworth CE
Primary School

A place of discovery and friendship

Review date: January 2013

Kibworth Primary School

Pencil and paper procedures

Background to the policy

This policy contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of the Primary National Strategy for mathematics. The mental methods in the *Primary Framework for teaching mathematics* will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop the necessary skills. However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at this school children will be encouraged to see mathematics as **both a written and spoken language**.

Teachers will support and guide children through the following important stages:

- developing the use of pictures and a mixture of words and symbols to represent numerical activities;
- using standard symbols and conventions;
- use of jottings to aid a mental strategy;
- use of pencil and paper procedures;
- use of expanded and compact calculation methods
- use of a calculator.

This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of expanded and compact calculation methods. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Children will always be encouraged to look at a calculation/problem and then decide which is the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator. Our long-term aim is for children to be able to select an efficient method of their choice (whether this be mental, written or using a calculator) that is appropriate for a given task. They will do this by always asking themselves:

‘Can I do this in my head?’

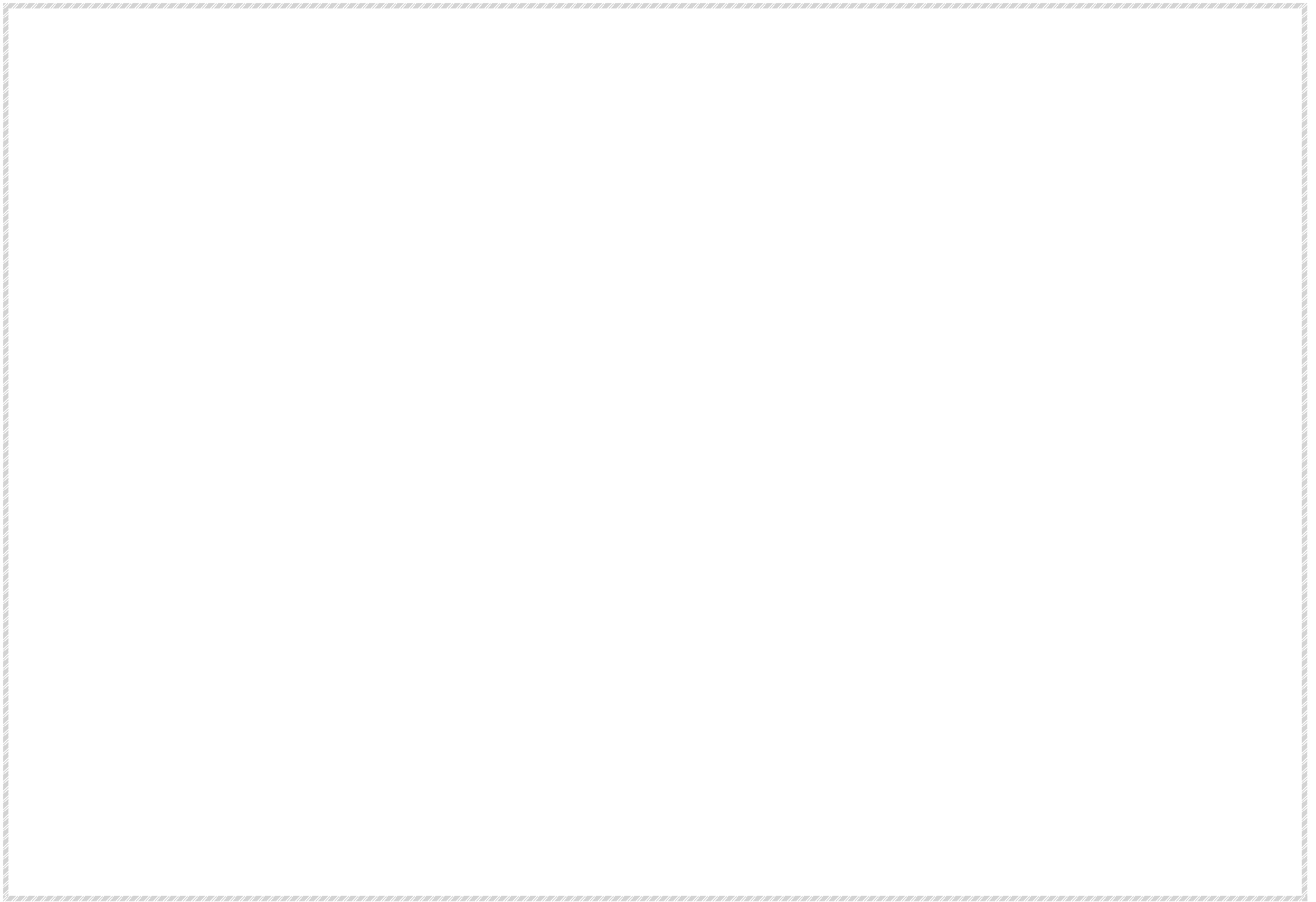
‘Can I use drawings or jottings to support my mental calculation?’

‘Do I need to use an expanded or compact calculation method?’

‘Do I need a calculator?’

The policy is written in stages that demonstrate a clear progression towards the development of formal written algorithm.

Children will only progress on to the next stage for each operation when they have mastered the pencil and paper procedures in their current stage.



Addition

Stage 1

Stage 2

Stage 3

<p>Addition Objectives</p> <p>PF-calculating Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number</p> <p><i>Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences</i></p> <p>PF-knowing and using number facts Derive and recall all pairs of numbers with a total of 10 and addition facts for totals to at least 5; work out the corresponding subtraction facts</p>	<p>Addition Objectives</p> <p>PF-calculating <i>Add or subtract mentally a one-digit number or a multiple of 10 to or from any two-digit number; use practical and informal written methods to add and subtract two-digit numbers</i></p> <p>Understand that subtraction is the inverse of addition and vice versa; use this to derive and record related addition and subtraction number sentences</p> <p><i>Use the symbols +, -, *, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. $\square \div 2 = 6$, $30 - \square = 24$)</i></p> <p>PF-knowing and using number facts Derive and recall all addition and subtraction facts for each number to at least 10, all pairs with totals to 20 and all pairs of multiples of 10 with totals up to 100</p>	<p>Addition Objectives</p> <p>PF-calculating Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers</p> <p><i>Add or subtract mentally combinations of one-digit and two-digit numbers</i></p> <p>PF-knowing and using number facts Derive and recall all addition and subtraction facts for each number to 20, sums and differences of multiples of 10 and number pairs that total 100</p>
---	---	--

REMEMBER: Children should have the opportunity to choose the appropriate strategy whether this is, mental strategies, mental with jottings, calculator or written procedures.

Addition

Stage 2

Stage 1

+ = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

$$2 = 1 + 1$$

$$2 + 3 = 4 + 1$$

$$3 = 3$$

$$2 + 2 + 2 = 4 + 2$$

Missing numbers need to be placed in all possible places.

$$3 + 4 = \square \qquad \square = 3 + 4$$

$$3 + \square = 7 \qquad 7 = \square + 4$$

$$\square + 4 = 7 \qquad 7 = 3 + \square$$

$$\square + \nabla = 7 \qquad 7 = \square + \nabla$$

Activities

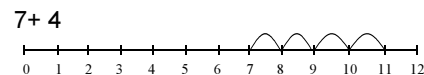
Children should have access to a wide range of counting equipment, everyday objects, as well as hoops, sorting trays, number tracks and numbered number lines.

Teacher modelling

Drawing jumps on numbered number lines to support understanding of the mental method

Children

To create their own jumps using rulers, fingers, pens, bodies etc.



Stage 3

+ = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

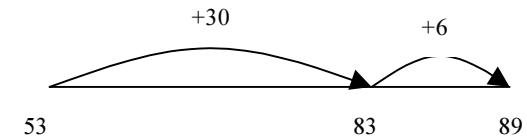
Partition into tens and ones

- Partition both numbers and recombine.
- Count on by partitioning the second number only e.g.

$$36 + 53 = 53 + 30 + 6$$

$$= 83 + 6$$

$$= 89$$

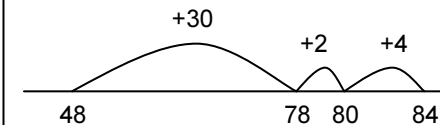


Add a near multiple of 10 to a two-digit number *Secure mental methods by using a number line to model the method.*

Continue as in Year 2 but with appropriate numbers e.g. $35 + 19$ is the same as $35 + 20 - 1$.

Children need to be secure adding multiples of 10 to any two-digit number including those that are not multiples of 10.

$$48 + 36 = 84$$



Expanded written method

$$83 + 42 = 125$$

either

1. Vertical expansion

$$\begin{array}{r} 83 \\ + 42 \\ \hline 120 \\ 5 \\ \hline 125 \end{array}$$

or

2. Horizontal expansion

$$\begin{array}{r} 80 + 3 \\ + 40 + 2 \\ \hline 120 + 5 = 125 \end{array}$$

Children should be able to make the choice of reverting to the expanded method if experiencing difficulty.

+ = signs and missing numbers

Continue using a range of equations as in Year 1 but with appropriate, larger numbers.

Extend to

$$14 + 5 = 10 + \square$$

and

$$32 + \square + \square = 100 \quad 35 = 1 + \square + 5$$

Partition in different ways and recombine

$$12 + 23 = 10 + 2 + 20 + 3$$

$$= 30 + 5$$

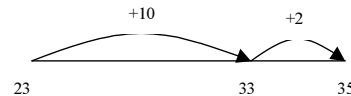
$$= 35$$

Count on in tens and ones

$$23 + 12 = 23 + 10 + 2$$

$$= 33 + 2$$

$$= 35$$

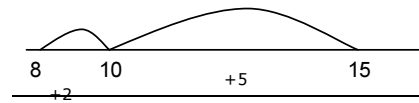


Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10 e.g.

Children should be able to partition the 7 to relate adding the 2 and then the 5.

$$8 + 7 = 15$$

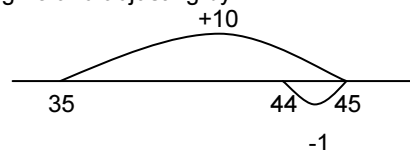


Add 9 or 11 by adding 10 and adjusting by 1

e.g.

Add 9 by adding 10 and adjusting by 1

$$35 + 9 = 44$$



Stage 4	Addition Stage 5	Stage 6
<p>Addition Objectives</p> <p>PF-calculating</p> <p>Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p</p> <p><i>Add or subtract mentally pairs of two-digit whole numbers (e.g. $47 + 58$, $91 - 35$)</i></p>	<p>Addition Objectives</p> <p>PF-calculating</p> <p><i>Use efficient written methods to add and subtract whole numbers and decimals with up to two places</i></p>	<p>Addition Objectives</p> <p>PF-calculating</p> <p><i>Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer</i></p>

REMEMBER: Children should have the opportunity to choose the appropriate strategy whether this is, mental strategies, mental with jottings, calculator or written procedures.

Addition

Stage 4

Stage 5

Stage 6

+ = signs and missing numbers

Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.

Add the nearest multiple of 10, then adjust

Continue as in Stage 2 and 3 but with appropriate numbers e.g. $63 + 29$ is the same as $63 + 30 - 1$

Towards a compact written method

$$367 + 185 = 431$$

either

$$\begin{array}{r} 367 \\ +185 \\ \hline 12 \\ 140 \\ \hline 400 \\ 552 \end{array}$$

or

$$\begin{array}{l} 300 + 60 + 7 \\ 100 + 80 + 5 \\ \hline 400 + 140 + 12 = 552 \end{array}$$

leading to

$$\begin{array}{r} 367 \\ +185 \\ \hline 552 \\ 100 \ 10 \end{array}$$

Extend to decimals in the context of money.

+ = signs and missing numbers

Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.

Add or subtract the nearest multiple of 10 or 100, then adjust

Continue as in Stage 2, 3 and 4 but with appropriate numbers e.g. $458 + 79 =$ is the same as $458 + 80 - 1$

Compact written method

Extend to numbers with at least four digits

$$3587 + 675 = 4262$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ 111 \end{array}$$

Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 1 \ 1 \end{array}$$

+ = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

Add the nearest multiple of 10, 100 or 1000, then adjust

Continue as in Stages 2, 3, 4 and 5 but with appropriate numbers including extending to adding 0.9, 1.9, 2.9 etc.

Compact written method

Extend to numbers with any number of digits and decimals with 1, 2 and/or 3 decimal places.

$$13.86 + 9.481 = 23.341$$

$$\begin{array}{r} 13.86 \\ + 9.481 \\ \hline 23.341 \\ 11 \ 1 \end{array}$$

Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.

REMEMBER: Children should have the opportunity to choose the appropriate strategy whether this is, mental strategies, mental with jottings, calculator or written procedures.

Subtraction

Year 1

Year 2

Year 3

<p>Subtraction Objectives</p> <p>PF-calculating Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number</p> <p><i>Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences</i></p>	<p>Subtraction Objectives</p> <p>PF-calculating <i>Add or subtract mentally a one-digit number or a multiple of 10 to or from any two-digit number; use practical and informal written methods to add and subtract two-digit numbers</i></p> <p>Understand that subtraction is the inverse of addition and vice versa; use this to derive and record related addition and subtraction number sentences</p> <p><i>Use the symbols +, -, *, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. $\square \div 2 = 6$, $30 - \square = 24$)</i></p>	<p>Subtraction Objectives</p> <p>PF-calculating Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers</p> <p><i>Add or subtract mentally combinations of one-digit and two-digit numbers</i></p> <p>PF-knowing and using number facts Derive and recall all addition and subtraction facts for each number to 20, sums and differences of multiples of 10 and number pairs that total 100</p>
--	--	---

REMEMBER: Children should have the opportunity to choose the appropriate strategy whether this is, mental strategies, mental with jottings, calculator or written procedures.

Subtraction

Stage 1

Children need to be given experience of all the strategies in a range of contexts, in order for them to make an informed choice of the most appropriate method to use when completing calculations.

Stage 2

Stage 3

- = signs and missing numbers

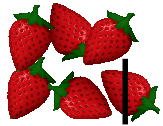
$$7 - 3 = \square \quad \square = 7 - 3$$

$$7 - \square = 4 \quad 4 = \square - 3$$

$$\square - 3 = 4 \quad 4 = 7 - \square$$

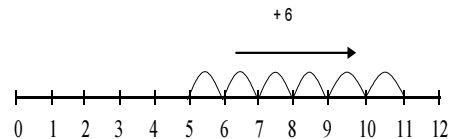
$$\square - \nabla = 4 \quad 4 = \square - \nabla$$

- Understand subtraction as 'take away'



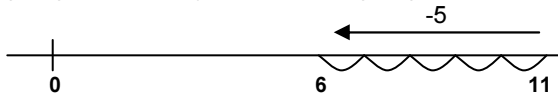
- Find a 'difference' by counting up;

I have saved 5p. The socks that I want to buy cost 11p. How much more do I need in order to buy the socks?



- Use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number.

I have 11 toy cars. There are 5 cars too many to fit in the garage. How many cars fit in the garage?



Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences

Recording by

- drawing jumps on prepared lines
- constructing own lines

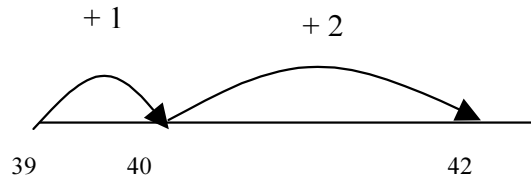
- = signs and missing numbers

Continue using a range of equations as in Stage 1 but with appropriate numbers.

Extend to $14 + 5 = 20 - \square$

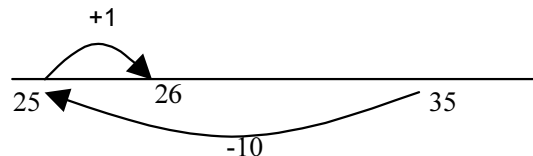
Find a small difference by counting up

$$42 - 39 = 3$$



Subtract 9 or 11. Begin to add/subtract 19 or 21

$$35 - 9 = 26$$



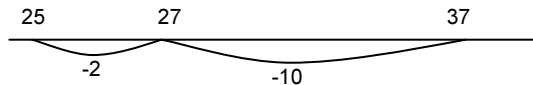
Use known number facts and place value to subtract

(partition second number only)

$$37 - 12 = 37 - 10 - 2$$

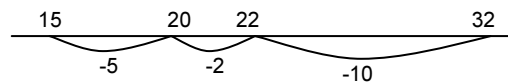
$$= 27 - 2$$

$$= 25$$



Bridge through 10 where necessary

$$32 - 17$$



- = signs and missing numbers

Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.

Find a small difference by counting up

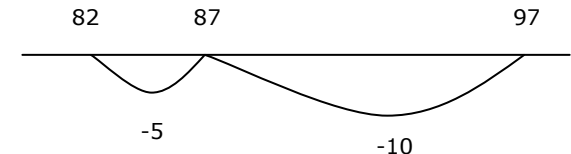
Continue as in Stage 2 but with appropriate numbers e.g. $102 - 97 = 5$

Subtract mentally a 'near multiple of 10' to or from a two-digit number

Continue as in Stage 2 but with appropriate numbers e.g. $78 - 49$ is the same as $78 - 50 + 1$

Use known number facts and place value to subtract

Continue as in Stage 2 but with appropriate numbers e.g. $97 - 15 = 72$



With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as $57 - 12$, $86 - 77$ or $43 - 28$.

Expanded written method without decomposition

$$168 - 54$$

$$100 + 60 + 8$$

$$- \quad \quad 50 + 4$$

$$100 + 10 + 4 = 114$$

Subtraction		
Stage 4	Stage 5	Stage 6
<p>Subtraction Objectives</p> <p>PF-calculating Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p</p> <p><i>Add or subtract mentally pairs of two-digit whole numbers (e.g. 47 + 58, 91 - 35)</i></p> <p>PF – Knowing & Using Number Facts Use knowledge of addition and subtraction facts and place value to derive sums and differences of pairs of multiples of 10, 100 or 1000</p>	<p>Subtraction Objectives</p> <p>PF-calculating <i>Use efficient written methods to add and subtract whole numbers and decimals with up to two places</i></p> <p>Extend mental-methods for whole-number calculations, for example to subtract one near-multiple of 1000 from another (e.g. 6070 - 4097)</p> <p>PF – Knowing & Using Number Facts <i>Use knowledge of place value and addition and subtraction of two-digit numbers to derive sums and differences of decimals (e.g. 6.5 ± 2.7, half of 5.6, double 0.34)</i></p>	<p>Subtraction Objectives</p> <p>PF-calculating <i>Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer</i></p> <p>Calculate mentally with integers and decimals: U.t ± U.t,</p>

REMEMBER: Children should have the opportunity to choose the appropriate strategy whether this is, mental strategies, mental with jottings, calculator or written procedures.

Subtraction

Stage 4

Stage 5

Stage 6

- = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

Find a small difference by counting up

e.g. $5003 - 4996 = 7$

This can be modelled on an empty number line. Children should be encouraged to use known number facts to reduce the number of steps.

Subtract the nearest multiple of 10, then adjust.

Continue as in Stages and 3 but with appropriate numbers.

Expanded written method (with and without decomposition)

$$\begin{array}{r} 100 + 70 + 8 \\ - \quad 40 + 7 \\ \hline 100 + 30 + 1 = 131 \end{array}$$

$$\begin{array}{r} 80 \quad 10+ \\ 100 + 90 + 2 \\ - \quad 50 + 7 \\ \hline 100 + 30 + 5 \end{array}$$

- = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

Find a difference by counting up

e.g. $8006 - 2993 = 5013$

This can be modelled on an empty number line.

Subtract the nearest multiple of 10 or 100, then adjust.

Continue as in Stages 2, 3 and 4 but with appropriate numbers.

Towards a compact written method with and without decomposition)

Without decomposition

$$\begin{array}{r} 7658 \\ -5342 \\ \hline 2316 \end{array}$$

With decomposition

$$\begin{array}{r} 410+ \\ 854 \\ -247 \\ \hline 607 \end{array}$$

Extend to decimals (tenths).

$$\begin{array}{r} 8.7 \\ -2.4 \\ \hline 6.3 \end{array} \quad \begin{array}{r} 5 \quad 10+ \\ 6.5 \\ -1.7 \\ \hline 4.8 \end{array}$$

- = signs and missing numbers

Continue using a range of equations as in Stages 1 and 2 but with appropriate numbers.

Find a difference by counting up

e.g. $8000 - 2785 = 5215$

To make this method more efficient, the number of steps should be reduced to a minimum through children knowing:

- Complements to 1, involving decimals to two decimal places ($0.16 + 0.84$)
- Complements to 10, 100 and 100

Subtract the nearest multiple of 10, 100 or 1000, then adjust

Continue as in Stages 2, 3, 4 and 5 but with appropriate numbers.

Compact written method

Continue to use a range of equations as in Stage 5 but with appropriate numbers.

$$\begin{array}{r} 4 \quad 12 \quad 13 \quad 10+ \\ 5345 \\ -3468 \\ \hline 1877 \end{array}$$

Extend to decimals (*hundredths*).

$$\begin{array}{r} 10+ \\ 12.42 \\ -4.31 \\ \hline 8.11 \end{array}$$

Multiplication		
Stage 1	Stage 2	Stage 3
<p>Multiplication Objectives</p> <p>PF-calculating Solve practical problems that involve combining groups of 2, 5 or 10, or sharing into equal groups</p> <p>PF-knowing & using number facts Count on or back in ones, twos, fives and tens and use this knowledge to derive the multiples of 2, 5 and 10 to the tenth multiple</p>	<p>Multiplication Objectives</p> <p>PF-calculating Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders</p> <p>PF-knowing & using number facts Derive and recall multiplication facts for the 2, 5 and 10 times-tables and the related division facts; recognise multiples of 2, 5 and 10</p>	<p>Multiplication Objectives</p> <p>PF-calculating Use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13×3, $50 \div 4$); round remainders up or down, depending on the context</p> <p>Multiply one-digit and two-digit numbers by 10 or 100, and describe the effect</p> <p>PF-knowing & using number facts Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables and the corresponding division facts; recognise multiples of 2, 5 or 10 up to 1000</p>

REMEMBER: Children should have the opportunity to choose the appropriate strategy whether this is, mental strategies, mental with jottings, calculator or written procedures.

Stage 1

Multiplication Stage 2

Stage 3

Multiplication is related to doubling and counting groups of the same size.



Looking at columns
 $2 + 2 + 2$
 3 groups of 2

Looking at rows
 $3 + 3$
 2 groups of 3

Counting using a variety of practical resources

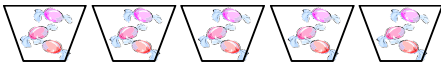
Counting in 2s e.g. counting socks, shoes, animal's legs...

Counting in 5s e.g. counting fingers, fingers in gloves, toes...

Counting in 10s e.g. fingers, toes...

Pictures / marks

There are 3 sweets in one bag.
 How many sweets are there in 5 bags?

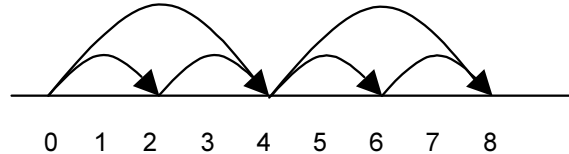


x = signs and missing numbers

$$\begin{array}{ll} 7 \times 2 = \square & \square = 2 \times 7 \\ 7 \times \square = 14 & 14 = \square \times 7 \\ \square \times 2 = 14 & 14 = 2 \times \square \\ \square \times \nabla = 14 & 14 = \square \times \nabla \end{array}$$

Arrays and repeated addition

$$\begin{array}{l} \bullet \bullet \bullet \bullet \quad 4 \times 2 \text{ or } 4 + 4 \\ \bullet \bullet \bullet \bullet \\ 2 \times 4 \text{ or } 2 + 2 + 2 + 2 \end{array}$$



Doubling multiples of 5 up to 50

$$15 \times 2 = 30$$

Partition

Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways: $6 = 5 + 1$ so e.g. Double 6 is the same as double five add double one.



$$\begin{array}{r} \text{AND double 15} \\ 10 \quad + \quad 5 \\ \downarrow \quad \quad \downarrow \times 2 \\ 20 \quad + \quad 10 = 30 \end{array}$$

Leading to

$$\begin{array}{r|l} \times & 10 \quad 5 \\ \hline 2 & 20 \quad 10 \quad = 30 \end{array}$$

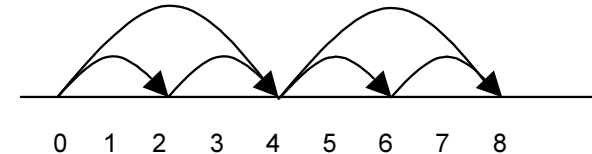
x = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers.

Arrays and repeated addition

Continue to understand multiplication as repeated addition and continue to use arrays (as in Stage 2).

$$\begin{array}{l} \bullet \bullet \bullet \bullet \bullet \quad 4 \times 2 \text{ or } 4 + 4 \\ \bullet \bullet \bullet \bullet \\ 2 \times 4 \text{ or } 2 + 2 + 2 + 2 \end{array}$$



Doubling multiples of 5 up to 50

$$35 \times 2 = 70$$

$$\begin{array}{r|l} \text{Partition} & \\ \hline \times & 30 \quad 5 \\ \hline 2 & 60 \quad 10 \quad = 70 \end{array}$$

Use known facts and place value to carry out simple multiplications

Use the same method as above (partitioning), e.g. $32 \times 3 = 96$

$$\begin{array}{r|l} \times & 30 & 2 \\ \hline 3 & 90 & 6 \\ & & = 96 \end{array}$$

Multiplication		
Stage 4	Stage 5	Stage 6
<p>Multiplication Objectives</p> <p>PF-calculating <i>Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15×9, $98 \div 6$)</i></p> <p>Multiply and divide numbers to 1000 by 10 and then 100 (whole-number answers), understanding the effect; relate to scaling up or down</p> <p>PF-Knowing and using number facts <i>Derive and recall multiplication facts up to 10×10, the corresponding division facts and multiples of numbers to 10 up to the tenth multiple</i></p>	<p>Multiplication Objectives</p> <p>PF-calculating Refine and use efficient written methods to multiply and divide HTU \timesU, TU \timesTU, U.t \timesU and HTU \divU</p> <p>PF-Knowing and using number facts Recall quickly multiplication facts up to 10×10 and use them to multiply pairs of multiples of 10 and 100; derive quickly corresponding division facts</p>	<p>Multiplication Objectives</p> <p>PF-calculating <i>Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer</i></p> <p>PF-Knowing and using number facts <i>Use knowledge of place value and multiplication facts to 10×10 to derive related multiplication and division facts involving decimals (e.g. 0.8×7, $4.8 \div 6$)</i></p>

REMEMBER: Children should have the opportunity to choose the appropriate strategy whether this is, mental strategies, mental with jottings, calculator or written procedures.

Multiplication

Stage 4

Stage 5

Stage 6

x = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers

Expanded grid method

Extend to multiplying by 7, 8 and 9

Grid method

23 x 7 is approximately 20 x 10 = 200

$$\begin{array}{c|c|c} \times & 20 & 3 \\ \hline 7 & 140 & 21 \end{array} = 161$$

x = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers

Expanded grid method

Extend to multiplying by two-digit numbers.

72 x 38 is approximately 70 x 40 = 2,800

$$\begin{array}{c|c|c} \times & 70 & 2 \\ \hline 30 & 2100 & 60 \\ \hline 8 & 560 & 16 \end{array} = \begin{array}{r} 2160 \\ \underline{576} \\ 2736 \end{array}$$

372 x 24 is approximately 400 x 20 = 8000

$$\begin{array}{c|c|c|c} \times & 300 & 70 & 2 \\ \hline 20 & 6000 & 1400 & 40 \\ \hline 4 & 1200 & 280 & 8 \end{array} = \begin{array}{r} 7440 \\ \underline{1488} \\ 8928 \end{array}$$

x = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers

Expanded grid method

Extend to decimals with up to two decimal places.

4.68 x 6 is approximately 5 x 6 = 30

$$\begin{array}{c|c|c|c} \times & 4 & 0.6 & 0.08 \\ \hline 6 & 24 & 3.6 & 0.48 \end{array} = \begin{array}{r} 24 \\ \underline{3.6} \\ \underline{0.48} \\ 28.08 \\ \underline{1} \end{array}$$

Expanded vertical written method

Extend to written method to cut down number of steps used to solve a calculation.

$$\begin{array}{r} 38 \\ \times 27 \\ \hline 760 \\ 210 \\ \hline 56 \\ \hline 1026 \\ \underline{1} \end{array}$$

REMEMBER: Children should have the opportunity to choose the appropriate strategy whether this is, mental strategies, mental with jottings, calculator or written procedures.

Stage 1	Division Stage 2	Stage 3
<p>PF - calculating Solve practical problems that involve combining groups of 2, 5 or 10, or sharing into equal groups</p> <p>PF-knowing & using number facts Count on or back in ones, twos, fives and tens and use this knowledge to derive the multiples of 2, 5 and 10 to the tenth multiple</p>	<p>Division Objectives (excluding rapid recall)</p> <p>PF - calculating Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders</p> <p>PF-knowing & using number facts Derive and recall multiplication facts for the 2, 5 and 10 times-tables and the related division facts; recognise multiples of 2, 5 and 10</p>	<p>Division Objectives (excluding rapid recall)</p> <p>PF - calculating Use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13×3, $50 \div 4$); round remainders up or down, depending on the context</p> <p>Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences</p> <p>Find unit fractions of numbers and quantities (e.g. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ of 12 litres)</p> <p>PF-knowing & using number facts Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables and the corresponding division facts; recognise multiples of 2, 5 or 10 up to 1000</p>

REMEMBER: Children should have the opportunity to choose the appropriate strategy whether this is, mental strategies, mental with jottings, calculator or written procedures.

Stage 1

Sharing

Requires secure counting skills
-see counting and understanding number strand
Develops importance of one-to-one correspondence
See appendix for additional information on x and ÷ and aspects of number

Sharing – 6 sweets are shared between 2 people. How many do they have each?



Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc.

Grouping

Sorting objects into 2s / 3s / 4s etc
How many pairs of socks are there?



There are 12 crocus bulbs. Plant 3 in each pot. How many pots are there?
Jo has 12 Lego wheels. How many cars can she make?

Division Stage 2

÷ = signs and missing numbers

$$\begin{array}{l} 6 \div 2 = \square \qquad \square = 6 \div 2 \\ 6 \div \square = 3 \qquad 3 = 6 \div \square \\ \square \div 2 = 3 \qquad 3 = \square \div 2 \\ \square \div \nabla = 3 \qquad 3 = \square \div \nabla \end{array}$$

Grouping

Link to counting and understanding number strand
Count up to 100 objects by grouping them and counting in tens, fives or twos;...

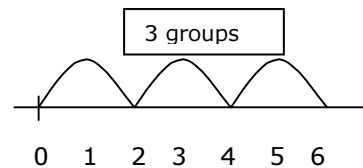
Find one half, one quarter and three quarters of shapes and sets of objects

6 ÷ 2 can be modelled as:

There are 6 strawberries.

How many people can have 2 each? How many 2s make 6?

6 ÷ 2 can be modelled as:



In the context of money count forwards and backwards using 2p, 5p and 10p coins

Practical grouping e.g. in PE

12 children get into teams of 4 to play a game. How many teams are there?



Stage 3

÷ = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers.

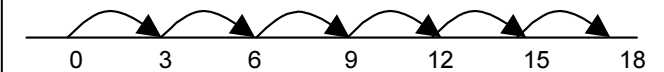
Understand division as sharing and grouping

18 ÷ 3 can be modelled as:

Sharing – 18 shared between 3 (see Stage 1 diagram)

OR

Grouping - How many 3's make 18?

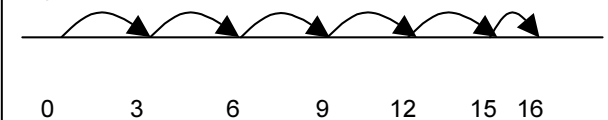


Remainders

16 ÷ 3 = 5 r1

Sharing - 16 shared between 3, how many left over?

Grouping – How many 3's make 16, how many left over?
e.g.



Expanded written method

Partition the dividend in to multiples of the divisor.

$$54 \div 3 = 18$$

$$\begin{array}{r} 30 + 24 \\ \downarrow \quad \downarrow \\ 10 + 8 = 18 \end{array} \div 3$$

Stage 4	Division Stage 5		Stage 6
<p>PF – calculating <i>Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15 × 9, 98 ÷ 6)</i></p> <p>PF-Knowing and using number facts <i>Derive and recall multiplication facts up to 10 x 10, the corresponding division facts and multiples of numbers to 10 up to the tenth multiple</i></p>	<p>Division Objectives</p> <p>PF – calculating Refine and use efficient written methods to multiply and divide HTU ×U, TU ×TU, U.t ×U and HTU ÷U</p> <p>PF-Knowing and using number facts Recall quickly multiplication facts up to 10 x 10 and use them to multiply pairs of multiples of 10 and 100; derive quickly corresponding division facts</p>	<p>Division Objectives</p> <p>PF – calculating <i>Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer</i></p>	

REMEMBER: Children should have the opportunity to choose the appropriate strategy whether this is, mental strategies, mental with jottings, calculator or written procedures.

Division Stage 5

Stage 4

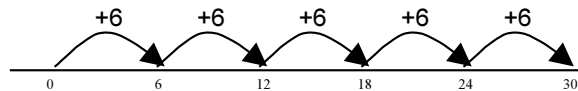
÷ = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers.

Sharing and grouping

30 ÷ 6 can be modelled as:

grouping – groups of 6 placed on no. line and the number of groups counted e.g.



sharing – sharing among 6, the number given to each person

Remainders

$$41 \div 4 = 10 \text{ r}1$$



$$41 = (10 \times 4) + 1$$

Expanded written method

As with Stage 3 but include divisors of 7, 8 and 9.

Expanded written method with remainders

$$72 \div 5 = 16\text{r}2$$

$$\begin{array}{r} 50 + 22 \\ \downarrow \quad \downarrow \quad \div 5 \end{array}$$

$$10 + 4\text{r}2 = 16\text{r}2$$

÷ = signs and missing numbers

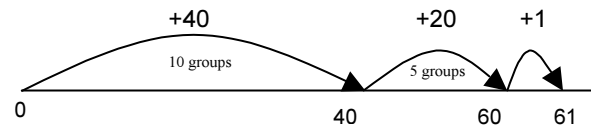
Continue using a range of equations as in Stage 2 but with appropriate numbers.

Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

Remainders

Quotients expressed as fractions or decimal fractions
 $61 \div 4 = 15 \frac{1}{4}$ or 15.25



Expanded written method

As with Stage 3 but include three-digit numbers as the dividend and calculations with remainders.

$$256 \div 7 = 36\text{r}4$$

$$\begin{array}{r} 210 + 46 \\ \downarrow \quad \downarrow \quad \div 7 \\ 30 + 6\text{r}4 = 36\text{r}4 \end{array}$$

Stage 6

÷ = signs and missing numbers

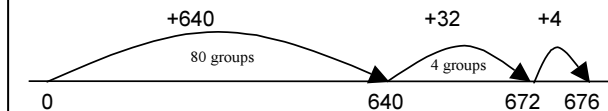
Continue using a range of equations as in Stage 2 but with appropriate numbers.

Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

Remainders

Quotients expressed as fractions or decimal fractions
 $676 \div 8 = 84.5$



Towards a compact written method

$$977 \div 36$$

Towards short division

From using the arrow method to find multiples of the divisor to short division.

- 1) Use the arrow method when dealing smaller numbers.

$$54 \div 3 = 18$$

$$\begin{array}{r} 30 + 24 \\ \downarrow \quad \downarrow \quad \div 3 \\ 10 + 8 = 18 \end{array}$$

- 2) Progress on to an expanded method of short division when numbers become trickier to divide.

$$162 \div 6 = 27$$

$$6 \overline{) 10 + 10 + 7 = 27} \quad \text{or} \quad 6 \overline{) 20 + 7 = 27}$$

(Note: In the original image, the numbers 60, 60, 42 and 120, 42 are written below the bars in the expanded methods.)

- 3) Learn the compact method for short division.

$$224 \div 7 = 32$$

$$\begin{array}{r} 32 \\ 7 \overline{) 224} \end{array}$$

We know that
 $7 \times 30 = 210$.
That leaves 14.