

# Arithmetic and mental strategies

**Counting forwards and backwards** Children first meet counting by beginning at one and counting on in ones. Their sense of number is extended by beginning at different numbers and counting forwards and backwards in steps, not only of ones, but also of twos, fives, tens, hundreds, tenths and so on. The image of a number line helps them to appreciate the idea of counting forwards and backwards. They will also learn that, when they add two numbers together, it is generally easier to count on from the larger number rather than the smaller. You will need to review children's 'counting on' strategies, then show them and encourage them to adopt more efficient methods.

**Reordering** Sometimes a calculation can be more easily worked out by changing the order of the numbers. The way in which children rearrange numbers in a particular calculation will depend on which number facts they can recall or derive quickly. It is important for children to know when numbers can be reordered: e.g.  $2 + 5 + 8 = 8 + 2 + 5$  or  $15 + 8 - 5 = 15 - 5 + 8$  or  $23 - 9 - 3 = 23 - 3 - 9$  and when they can't be reordered: e.g.  $8 - 5 \neq 5 - 8$  The strategy of changing the order of numbers applies mainly when the question is written down. It is more difficult to reorder numbers if the question is presented orally

**Partitioning: counting on or back** It is important for children to know that numbers can be partitioned into, for example, hundreds, tens and ones, so that  $326 = 300 + 20 + 6$ . In this way, numbers are seen as wholes, rather than as a collection of single digits in columns. This way of partitioning numbers can be a useful strategy for adding and subtracting pairs of numbers. Both numbers can be partitioned, although it is often helpful to keep the first number as it is and to partition just the second number.

**Partitioning: bridging through multiples of 10** An important aspect of having an appreciation of number is to know how close a number is to the next or the previous multiple of 10: to recognise, for example, that 47 is 3 away from 50, or that 47 is 7 away from 40. In mental addition or subtraction, it is often useful to count on or back in two steps, bridging a multiple of 10. The empty number line, with multiples of 10 as 'landmarks', is helpful, since children can visualise jumping to them. For example,  $6 + 7$  is worked out in two jumps, first to 10, then to 13

**Partitioning: compensating** This strategy is useful for adding and subtracting numbers that are close to a multiple of 10, such as numbers that end in 1 or 2, or 8 or 9. The number to be added or subtracted is rounded to a multiple of 10 plus or minus a small number. For example, adding 9 is carried out by adding 10, then subtracting 1; subtracting 18 is carried out by subtracting 20, then adding 2. A similar strategy works for adding or subtracting decimals that are close to whole numbers. For example:  $1.4 + 2.9 = 1.4 + 3 - 0.1$  or  $2.45 - 1.9 = 2.45 - 2 + 0.1$ .

**Partitioning: using 'near' doubles** If children have instant recall of doubles, they can use this information when adding two numbers that are very close to each other. So, knowing that  $6 + 6 = 12$ , they can be encouraged to use this to help them find  $7 + 6$ , rather than use a counting on strategy or bridging through 10.

**Partitioning: bridging through 60** to calculate a time interval Time is a universal non-metric measure. A digital clock displaying 9.59 will, in two minutes time, read 10.01 not 9.61. When children use minutes and hours to calculate time intervals, they have to bridge through 60. So to find the time 20 minutes after 8.50am, for example, children might say 8.50am plus 10 minutes takes us to 9.00am, then add another 10 minutes

**Eyfs- Mental Maths end of Year expectations**

- **Subitise numbers up to 5**
- **To know 1 more/1 less than a number up to 10**
- **Automatically recall number bonds to 5**
- **Automatically recall SOME number bonds to 10**
- **Automatically recall SOME double facts**

Year 1	Autumn 1	Autumn 2	Spring 1	Spring 2	Summer 1	Summer 2
Addition and subtraction	<ul style="list-style-type: none"> <li>• To partition numbers 3, 4, 5, 6</li> <li>• Adding and subtracting 1 within 10</li> <li>• Double numbers to 5 (e.g. 4 + 4)</li> <li>• Number bonds to ten (e.g. 2+8 and 8+2)</li> </ul>	<ul style="list-style-type: none"> <li>• Adding and subtracting 2 within 10</li> <li>• Partition numbers 7, 8, 9</li> </ul>	<ul style="list-style-type: none"> <li>• represent and use number bonds and related subtraction facts within 20</li> <li>• add and subtract one-digit and two-digit numbers to 20, including zero</li> </ul>	<ul style="list-style-type: none"> <li>• Place value to 50.</li> </ul>	<ul style="list-style-type: none"> <li>• Adding 10 to a number (e.g. 5 + 10 and 10+5)</li> </ul>	<ul style="list-style-type: none"> <li>• Near doubles ( e.g. 3 _ 4 and 4+ 3)</li> </ul>
Multiplication and division				Count in <b>multiples of 5</b> up to 60, linking with knowledge of counting in 10s.		Count in <b>multiples of 10, 2 and 5</b> in order with growing fluency.

				<p>Continue to develop fluency of <b>counting</b> in 2s and 10s. <b>Count in multiples of 2</b> up to 24, linking with <b>even numbers</b> and supporting <b>doubles</b>.</p> <p><b>Count in multiples of 10</b> in order up to 120.</p>		<p><b>Count in multiples of 10, 2 and 5</b> in order fluently.</p> <p><b>Begin to recall multiples of 10</b> up to 12x10 with growing fluency.</p>
Double and half				Double all numbers to 10, e.g. double 9		
Mental Strategies to use	<p>Counting forwards and backwards to 20. Then move to 50. Then to 100.</p> <p>Being able to subitise numbers to 10 i.e knowing that <math>3 + 2 = 5</math>.</p> <ul style="list-style-type: none"> <li>• reorder numbers when adding, e.g. put the larger number first</li> <li>• count on or back in ones, twos or tens</li> <li>• partition small numbers, e.g. <math>8 + 3 = 8 + 2 + 1</math></li> <li>• partition and combine tens and ones <ul style="list-style-type: none"> <li>• partition: double and adjust, e.g. <math>5 + 6 = 5 + 5 + 1</math></li> </ul> </li> </ul>					

Example calculations	Possible counting strategy
$4 + 5$	count on in ones from 4 (or in ones from 5)
$8 - 3$	count back in ones from 8
$10 + 7$	count on in ones from 10 (or use place value)
$13 + 5$	count on in ones from 13
$17 - 3$	count back in ones from 17
$18 - 6$	count back in twos

Example calculations	Possible reordering strategy
$2 + 7$	$7 + 2$
$5 + 13$	$13 + 5$
$10 + 2 + 10$	$10 + 10 + 2$

Present children with groups of three then four numbers that they are to add in their head. Make sure that, in each group of numbers, there are two numbers that have a total of 10. For example:

$$8 + 3 + 5 + 2$$

Discuss their methods. See if any children chose to add  $8 + 2$  first and then add on the  $5 + 3$ , or linked the  $3 + 5$  and added  $8 + (3 + 5) + 2$ .

Partitioning: using 'near' doubles

Example calculations	Possible compensation strategy
$6 + 7$	is double 6 and add 1 or double 7 and subtract 1

## Year 2

Year 2      Autumn 1      Autumn 2      Spring 1      Spring 2      Summer 1      Summer 2

Addition and subtraction (mentally)

- Review number bonds to 10
  - Review adding and subtracting to 10
  - Partition numbers 11-20
  - addition and subtraction facts for all numbers to 20, e.g.  $9 + 8$ ,  $17 - 9$ , drawing on knowledge of inverse operations
  - Number bonds with 20 deriving from number bonds to 10
  - Use related number bonds for number bonds to 100
- add or subtract a two-digit number to or from a multiple of 10, e.g.  $50 + 38$ ,  $90 - 27$
  - Doubles to 10 (e.g.  $7+7$ )
  - add a groups of small numbers e.g.  $2 + 8 + 7$
  - Near Doubles (e.g.  $5+6$  and  $6+5$ )
  - Bridging tens (e.g.  $8+4$ )
  - Compensating

<p><b>Written addition and subtraction</b></p>	<p>§ add and subtract numbers using concrete objects, pictorial representations, and mentally, including:          § a two-digit number and ones          § a two-digit number and tens          § two two-digit numbers          § adding three one-digit numbers</p>				<ul style="list-style-type: none"> <li>•</li> </ul>	<ul style="list-style-type: none"> <li>•</li> </ul>
<p><b>Multiplication and division</b></p>			<p>Consolidate <b>counting</b> in <b>multiples of 2, 5 and 10</b> in order from 0 up to 12x.</p> <p><b>Recall multiples of 10</b> up to 12x10 fluently, in any order.</p> <p><b>Count</b> in <b>multiples of 2 and 5</b> from 0 up to 12x fluently.</p> <p><b>Recall multiples of 10</b> up to 10 x 12 in any order, including missing numbers and related division facts with growing fluency.</p> <p><b>Recall multiples of 2</b> up to 2 x 12 fluently, in any order</p>	<p><b>Recall multiples of 5</b> up to 5 x 12 fluently, in any order.</p> <p><b>Recall multiples of 2</b> up to 2 x 12 in any order, including missing numbers and related division facts with growing fluency.</p>	<p><b>Count in multiples of 3</b> to 3 x 12 in order from 0 with growing fluency.</p> <p><b>Recall multiples of 2</b> up to 2 x 12 in any order, including missing numbers and related division facts fluently.</p> <p><b>Recall multiples of 5</b> up to 5 x 12 in any order, including missing numbers and related division facts</p>	<p><b>Count in multiples of 3</b> to 3 x 12 in order from 0 with fluency.</p> <p><b>Begin to recall multiples of 3</b> up to 3 x 12 in any order, including missing numbers.</p> <p>.</p>

			Recap <b>multiples of 10</b> up to 10 x 12 fluently.		with growing fluency.	
			Teaching of times table facts and methods, including <b>arrays</b> .			
Fractions				<ul style="list-style-type: none"> <li>recognise, find, name and write fractions <math>\frac{1}{3}</math>, <math>\frac{1}{4}</math>, <math>\frac{2}{4}</math>, <math>\frac{3}{4}</math> of a quantity</li> <li>Find 1 2, 1 3, 1 4, 1 5 and 1 10 of numbers</li> <li>Find half of any even number to 40 or multiple of 10 to 100, e.g. halve 80 Find half of any multiple of 10 up to 200, e.g. halve 170</li> </ul>		
Double and half		<ul style="list-style-type: none"> <li>Double all numbers to 20 and find the corresponding halves, e.g. double 7, half of 14</li> <li>Double multiples of 10 to 50, e.g. double 40, and find the corresponding halves</li> <li>Double multiples of 5 to 50 and find the corresponding halves, e.g. double 35, half of 70</li> </ul>				
Mental Strategies to use	<ul style="list-style-type: none"> <li>reorder numbers when adding</li> <li>partition: bridge through 10 and multiples of 10 when adding and subtracting</li> <li><b>partition</b> and combine multiples of tens and ones</li> <li>use knowledge of pairs making 10</li> <li><b>partition</b>: count on in tens and ones to find the total</li> <li><b>partition</b>: count on or back in tens and ones to find the difference</li> </ul>					



- **partition**: add a multiple of 10 and adjust by 1
- **partition**: double and adjust

Example calculations	Possible partitioning and counting strategy
30 + 47	30 + 40 + 7
78 - 40	70 + 8 - 40 = 70 - 40 + 8
17 + 14	10 + 7 + 10 + 4 = 10 + 10 + 7 + 4
23 + 45	40 + 5 + 20 + 3 = 40 + 20 + 5 + 3
68 - 32	60 + 8 - 30 - 2 = 60 - 30 + 8 - 2
55 + 37	55 + 30 + 7 = 85 + 7

Partitioning: bridging through multiples of 10

Example calculations	Possible bridging strategy
5 + 8 or 12 - 7	5 + 5 + 3 or 12 - 2 - 5
65 + 7 or 43 - 6	65 + 5 + 2 or 43 - 3 - 3
24 - 19	19 + 1 + 4
49 + 32	49 + 1 + 31
90 - 27	27 + 3 + 60

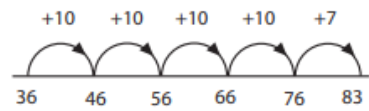
Partitioning: compensating

Example calculations	Possible compensation strategy
34 + 9 34 + 19 34 + 29 and so on	34 + 10 - 1 34 + 20 - 1 34 + 30 - 1 and so on
34 + 11 34 + 21 34 + 31 and so on	34 + 10 + 1 34 + 20 + 1 34 + 30 + 1 and so on
70 - 9	70 - 10 + 1
53 + 12	53 + 10 + 2
53 - 12	53 - 10 - 2
53 + 18	53 + 20 - 2
84 - 18	84 - 20 + 2

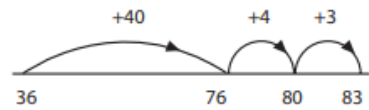
13 + 14	is double 13 and add 1 or double 14 and subtract 1
39 + 40	is double 40 and subtract 1
18 + 16	is double 18 and subtract 2 or double 16 and add 2
60 + 70	is double 60 and add 10 or double 70 and subtract 10

## Partitioning: using 'near' doubles

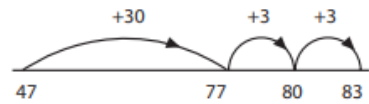
Get children to record a two-digit number on an empty number line. For example,  $36 + 47$  might be seen as counting on from 36 initially in steps of 10:



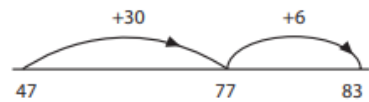
or by first counting on a step of 40 to 76, then bridging through 80 using two steps:



or by reordering the calculation and then counting on from 47, bridging through 80 using two steps:



or by counting on 30 to 77, then using knowledge of number facts to 20 and place value to reach 83 in one step:



## Reordering:

$5 + 34$	$34 + 5$
$5 + 7 + 5$	$5 + 5 + 7$
$23 + 54$	$54 + 23$
$12 - 7 - 2$	$12 - 2 - 7$
$13 + 21 + 13$	$13 + 13 + 21$ (using double 13)

## Partitioning: Counting on or back

$30 + 47$	$30 + 40 + 7$
$78 - 40$	$70 + 8 - 40 = 70 - 40 + 8$
$17 + 14$	$10 + 7 + 10 + 4 = 10 + 10 + 7 + 4$
$23 + 45$	$40 + 5 + 20 + 3 = 40 + 20 + 5 + 3$
$68 - 32$	$60 + 8 - 30 - 2 = 60 - 30 + 8 - 2$
$55 + 37$	$55 + 30 + 7 = 85 + 7$

Counting on or counting back:

<b>Year 2</b>	$23 + 5$	count on in ones from 23	$50 + 38$	count on in tens then ones from 50
	$57 - 3$	count back in ones from 57	$90 - 27$	count back in tens then ones from 90
	$60 + 5$	count on in ones from 60 (or use place value)	$34 + 65$	count on in tens then ones from 34
	$80 - 7$	count back in ones from 80 (or use knowledge of number facts to 10 and place value)	$87 - 23$	count back in tens then ones from 87
	$27 + 60$	count on in tens from 27	$35 + 15$	count on in steps of 5 from 35
	$72 - 50$	count back in tens from 72		

## Year 3

Year 3	Autumn 1	Autumn 2	Spring 1	Spring 2	Summer 1	Summer 2
Addition and subtraction (mentally)	<ul style="list-style-type: none"> <li>Sums and differences of multiples of 10, e.g. <math>50 + 80</math>, <math>120 - 90</math></li> <li>Review - add or subtract a two-digit number to or from a multiple of 10, e.g. <math>50 + 38</math>, <math>97 - 20</math></li> <li>Add and subtract two-digit numbers e.g. <math>34 + 65</math>, <math>68 - 35</math></li> <li>And a three-digit number and ones</li> <li>a three-digit number and tens</li> </ul>	<ul style="list-style-type: none"> <li>addition doubles for multiples of 10 to 100, e.g. <math>90 + 90</math></li> <li>Pairs of two-digit numbers with a total of 100, e.g. <math>32 + 68</math>, or <math>32 + \underline{\quad} = 100</math></li> <li>Solving Problems, including missing number problems.</li> </ul>	<ul style="list-style-type: none"> <li></li> </ul>	<ul style="list-style-type: none"> <li></li> </ul>	<ul style="list-style-type: none"> <li>add near doubles, e.g. <math>18 + 16</math>, <math>60 + 70</math></li> <li></li> </ul>	<ul style="list-style-type: none"> <li>Adding and subtracting time amounts mentally using counting on and bridging.</li> <li>Add and subtract money.</li> </ul>

	<ul style="list-style-type: none"> <li>a three-digit number and hundreds</li> </ul>					
Written addition and subtraction	<ul style="list-style-type: none"> <li>Add and subtract numbers with up to three digits, using formal written methods of column addition and subtraction</li> <li>solve problems, including missing number problems</li> </ul>				<ul style="list-style-type: none"> <li></li> </ul>	<ul style="list-style-type: none"> <li></li> </ul>
Multiplication and division (mental)	<p>Consolidate the 2, 5 and 10 times table.</p> <p><b>Understand</b> how to move one place value and use zero as a place holder to <b>multiply and divide by 10</b>.</p> <p>Continue to <b>count</b> in <b>multiples of 3 and 4</b> to 3 x 12 in order from 0 fluently.</p> <p><u>Understand</u> that anything <u>multiplied by zero</u> is zero.</p>	<p>Introduce and begin to <b>count</b> in <b>multiples of 3 and 4</b> from 0 to 12 using arrays and relating to 2 times table.</p>	<p><b>Recall multiples of 3</b> from 3 x 0 up to 3 x 12 in any order, including missing numbers and related division facts fluently.</p> <p><b>Recognise</b> any <b>multiple of 3</b> (all multiples of 3 have a digital root of 3, 6 or 9).</p> <p><b>Recall multiples of 4</b> from 4 x 0 up to 4 x 12 in any order, including missing numbers and related division facts with growing fluency.</p>	<p>Introduce and begin to <b>count</b> in <b>multiples of 8 and 6</b> from 0 to 12 using arrays and relating to 2, 4 and 3 times tables.</p>	<p><b>Recognise</b> that doubling and doubling again is <b>equivalent to multiplying by 4</b>.</p> <p>Fluently <b>count</b> in <b>multiples of 8 and 6</b> in order up to x12, using multiples of 2, 4 and 3 to support</p> <p><b>Recognise</b> that <b>even multiples of 3</b> are also <b>multiples of 6</b>.</p>	<p><b>Recall multiples of 8</b> from 8 x 0 up to 8 x 12 in any order, including missing numbers and related division facts fluently.</p> <p><b>Recall multiples of 6</b> from 6 x 0 to 6 x 12 in any order, including missing numbers and related division facts with growing fluency</p>

	<p><u>Understand that multiplied by 1 stays the same.</u></p>					
Written multiplication		<ul style="list-style-type: none"> <li>• Two by one digit multiplication</li> <li>• Solve problems missing number problems, involving multiplication and division.</li> </ul>				
Fractions				<ul style="list-style-type: none"> <li>• recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators</li> <li>• Find unit fractions and simple non-unit fractions of whole numbers or quantities, e.g. <math>\frac{3}{8}</math> of 24</li> </ul>	<ul style="list-style-type: none"> <li>• add and subtract fractions with the same denominator within one whole</li> </ul>	
Double and half		<ul style="list-style-type: none"> <li>• Double multiples of 10 to 100, e.g. double 90, and corresponding halves</li> <li>• Double multiples of 5 to 100 and find the corresponding halves, e.g. double 85, halve 170</li> </ul>			<ul style="list-style-type: none"> <li>•</li> </ul>	
Mental Strategies to use	<ul style="list-style-type: none"> <li>• reorder numbers when adding</li> <li>• identify pairs totalling 10 or multiples of 10</li> <li>• partition: add tens and ones separately, then recombine</li> <li>• partition: count on in tens and ones to find the total</li> <li>• partition: count on or back in tens and ones to find the difference</li> </ul>					

- partition: add or subtract 10 or 20 and adjust
- partition: double and adjust
- partition: count on or back in minutes and hours, bridging through 60 (analogue times)

### Counting forwards and backwards

$50 + 38$	count on in tens then ones from 50	$73 - 68$	count up from 68, counting 2 to 70 then 3 to 73
$90 - 27$	count back in tens then ones from 90	$47 + 58$	count on 50 from 47, then 3 to 100, then 5 to 105
$34 + 65$	count on in tens then ones from 34	$124 - 47$	count back 40 from 124, then 4 to 80, then 3 to 77
$87 - 23$	count back in tens then ones from 87	$570 + 300$	count on in hundreds from 570
$35 + 15$	count on in steps of 5 from 35	$960 - 500$	count back in hundreds from 960

### Reordering:

		$6 + 13 + 4 + 3$	$6 + 4 + 13 + 3$
		$17 + 9 - 7$	$17 - 7 + 9$
		$28 + 75$	$75 + 28$ (thinking of 28 as 25 + 3)
$23 + 45$	$40 + 5 + 20 + 3 = 40 + 20 + 5 + 3$	$12 + 17 + 8 + 3$	$12 + 8 + 17 + 3$
$68 - 32$	$60 + 8 - 30 - 2 = 60 - 30 + 8 - 2$	$25 + 36 + 75$	$25 + 75 + 36$
$55 + 37$	$55 + 30 + 7 = 85 + 7$	$58 + 47 - 38$	$58 - 38 + 47$
$365 - 40$	$300 + 60 + 5 - 40 = 300 + 60 - 40 + 5$	$200 + 567$	$567 + 200$

### Partitioning: counting on or back

$23 + 45$	$40 + 5 + 20 + 3 = 40 + 20 + 5 + 3$
$68 - 32$	$60 + 8 - 30 - 2 = 60 - 30 + 8 - 2$
$55 + 37$	$55 + 30 + 7 = 85 + 7$
$365 - 40$	$300 + 60 + 5 - 40 = 300 + 60 - 40 + 5$

Partitioning: bridging through multiples of 10

$49 + 32$	$49 + 1 + 31$
$90 - 27$	$27 + 3 + 60$
$57 + 34$ or $92 - 25$	$57 + 3 + 31$ or $92 - 2 - 20 - 3$
$84 - 35$	$35 + 5 + 40 + 4$

Partitioning: compensating

$53 + 12$	$53 + 10 + 2$
$53 - 12$	$53 - 10 - 2$
$53 + 18$	$53 + 20 - 2$
$84 - 18$	$84 - 20 + 2$
$38 + 68$	$38 + 70 - 2$
$95 - 78$	$95 - 80 + 2$
$58 + 32$	$58 + 30 + 2$
$64 - 32$	$64 - 30 - 2$

Partitioning: using 'near' doubles

$18 + 16$	is double 18 and subtract 2 or double 16 and add 2
$60 + 70$	is double 60 and add 10 or double 70 and subtract 10
$76 + 75$	is double 76 and subtract 1 or double 75 and add 1



Partitioning: bridging through 60 to calculate a time interval

<b>Year 3</b>	It is 10.30am. How many minutes to 10.45am?
	It is 3.45pm. How many minutes to 4.15pm?

## Year 4

Year 4	Autumn	Spring	Summer				
Addition and subtraction mental	<b>Recap skills of year 3</b> <ul style="list-style-type: none"> <li>sums and differences of pairs of multiples of 10, 100 or 1000</li> </ul>	<ul style="list-style-type: none"> <li>addition doubles of numbers 1 to 100, e.g. <math>38 + 38</math>, and the corresponding halves</li> <li>what must be added to any three-digit number to make the next multiple of 100, e.g. <math>521 + \dots = 600</math></li> </ul>	<ul style="list-style-type: none"> <li>add or subtract a near multiple of 10, e.g. <math>56 + 29</math>, <math>86 - 38</math></li> <li>add near doubles of two digit numbers, e.g. <math>38 + 37</math></li> <li>add or subtract two-digit or three-digit multiples of 10, e.g. <math>120 - 40</math>, <math>140 + 150</math>, <math>370 - 180</math></li> </ul>				
Addition and subtraction written		add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate					
Multiplication and division	Recall multiples of 3, 4 and 8 up to x12 in any	Recall multiples of 6 in any order, including missing	Recall multiples of 7 in any	Recall multiples of 3,	Recall multiples of	Recall multiples of all times tables up	

	order, including missing numbers and related division facts fluently.	<p>numbers and related division facts with growing fluency.</p> <p><b>Recall multiples of 7</b> in any order, including missing numbers and related division facts with growing fluency.</p> <p>Learn 'tricky' facts using mnemonics e.g. "7 x 7 = 49, one short of 50 all the time."</p>	order, including missing numbers and related division facts with growing fluency.	<b>4, 6, 7, 8 and 9</b> up to x12 in any order, including missing numbers and related division facts fluently.	<b>11</b> in any order, including missing numbers and related division facts fluently (using 10x and adjusting by adding 1 more group).	to x12 in any order, including missing numbers and related division facts fluently.
Double and halving		<p>Double any two-digit number and find the corresponding halves, e.g. double 47, half of 94</p> <p>Double multiples of 10 and 100 and find the corresponding halves, e.g. double 800, double 340, half of 1600, half of 680</p>				
Written multiplication and division		<ul style="list-style-type: none"> <li>• use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers</li> <li>• multiply two-digit and three-digit numbers by a one-digit</li> </ul>		•		

		number using formal written layout				
Fractions			<ul style="list-style-type: none"> <li>• add and subtract fractions with the same denominator</li> <li>• Find half of any even number to 200 Find unit fractions and simple non-unit fractions of whole numbers or quantities, e.g. <math>\frac{3}{8}</math> of 24</li> </ul>	<ul style="list-style-type: none"> <li>• find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths</li> <li>• Recall fraction and decimal equivalents for one-half, quarters, tenths and hundredths, e.g. recall the equivalence of 0.3 and <math>\frac{3}{10}</math>, and 0.03 and <math>\frac{3}{100}</math></li> </ul>		
Mental strategies	<ul style="list-style-type: none"> <li>• count on or back in hundreds, tens and ones</li> <li>• partition: add tens and ones separately, then recombine</li> <li>• partition: subtract tens and then ones, e.g. subtracting 27 by subtracting 20 then 7</li> <li>• subtract by counting up from the smaller to the larger number • partition: add or subtract a multiple of 10 and adjust, e.g. <math>56 + 29 = 56 + 30 - 1</math>, or <math>86 - 38 = 86 - 40 + 2</math></li> <li>• partition: double and adjust</li> <li>• use knowledge of place value and related calculations, e.g. work out <math>140 + 150 = 290</math> using <math>14 + 15 = 29</math></li> <li>• partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)</li> </ul>					

Counting forwards and backwards

$73 - 68$	count up from 68, counting 2 to 70 then 3 to 73
$47 + 58$	count on 50 from 47, then 3 to 100, then 5 to 105
$124 - 47$	count back 40 from 124, then 4 to 80, then 3 to 77
$570 + 300$	count on in hundreds from 570
$960 - 500$	count back in hundreds from 960

### Reordering

$6 + 13 + 4 + 3$	$6 + 4 + 13 + 3$
$17 + 9 - 7$	$17 - 7 + 9$
$28 + 75$	$75 + 28$ (thinking of 28 as $25 + 3$ )
$12 + 17 + 8 + 3$	$12 + 8 + 17 + 3$
$25 + 36 + 75$	$25 + 75 + 36$
$58 + 47 - 38$	$58 - 38 + 47$
$200 + 567$	$567 + 200$

### Partitioning: counting on or back

$55 + 37$	$55 + 30 + 7 = 85 + 7$
$365 - 40$	$300 + 60 + 5 - 40 = 300 + 60 - 40 + 5$
$43 + 28 + 51$	$40 + 3 + 20 + 8 + 50 + 1 = 40 + 20 + 50 + 3 + 8 + 1$

### Partitioning: bridging through multiples of 10

$57 + 34$ or $92 - 25$	$57 + 3 + 31$ or $92 - 2 - 20 - 3$
$84 - 35$	$35 + 5 + 40 + 4$
$607 - 288$	$288 + 12 + 300 + 7$

### Partitioning: compensating

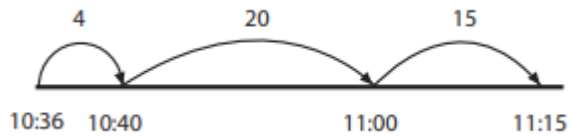
$38 + 68$	$38 + 70 - 2$
$95 - 78$	$95 - 80 + 2$
$58 + 32$	$58 + 30 + 2$
$64 - 32$	$64 - 30 - 2$
$138 + 69$	$138 + 70 - 1$
$405 - 399$	$405 - 400 + 5$

Partitioning: using 'near' doubles

$76 + 75$	is double 76 and subtract 1 or double 75 and add 1
$160 + 170$	is double 150, then add 10, then add 20 or double 160 and add 10 or double 170 and subtract 10

Partitioning: bridging through 60 to calculate a time interval

I get up 40 minutes after 6.30am. What time is that?
What is the time 50 minutes before 1.10pm?
It is 4.25pm. How many minutes to 5.05pm?



Year 5	Autumn	Spring	Summer
Addition and subtraction (mental)	<ul style="list-style-type: none"> <li>Recap Year 4</li> <li>what must be added to any four-digit number to make the next multiple of 1000, e.g. <math>4087 + \diamond = 5000</math></li> <li>add or subtract a pair of two digit numbers or three-digit multiples of 10, e.g. <math>38 + 86</math>, <math>620 - 380</math>, <math>350 + 360</math></li> <li>add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number, e.g. <math>235 + 198</math></li> <li>find the difference between near multiples of 100, e.g. <math>607 - 588</math>, or of 1000, e.g. <math>6070 - 4087</math></li> </ul>	<ul style="list-style-type: none"> <li>sums and differences of decimals, e.g. <math>6.5 + 2.7</math>, <math>7.8 - 1.3</math></li> <li>doubles and halves of decimals, e.g. half of 5.6, double 3.4</li> <li>what must be added to a decimal with units and tenths to make the next whole number, e.g. <math>7.2 + \diamond = 8</math></li> <li>add or subtract any pairs of decimal fractions each with units and tenths, e.g. <math>5.7 + 2.5</math>, <math>6.3 - 4.8</math></li> <li>Adding or subtracting decimals from two different place value columns where there are place holders. E.g. <math>4.02 + 1.6</math></li> </ul>	
Addition and subtraction	<ul style="list-style-type: none"> <li>Add and subtract numbers greater than 4 digits</li> </ul>		
Multiplication and division	<p>Recall <b>multiples of 12</b> in any order, including missing numbers and related division facts fluently.</p> <p>Recall <b>multiples of all times tables</b> up to <math>12 \times 12</math> in any order, including missing numbers and related division facts with fluency</p> <ul style="list-style-type: none"> <li>squares to <math>12 \times 12</math></li> </ul>	<ul style="list-style-type: none"> <li>division facts corresponding to tables up to <math>10 \times 10</math>, and the related unit fractions, e.g. <math>7 \times 9 = 63</math> so one-ninth of 63 is 7 and one-seventh of 63 is 9</li> <li>percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths</li> <li>factor pairs to 100</li> <li>Cube numbers</li> <li>Multiply by 10 and halve to multiply large numbers by 5 <ul style="list-style-type: none"> <li>Double and double again to <b>multiply large numbers by 4</b>.</li> </ul> </li> </ul> <p>Multiply by 10 and subtract one group to <b>multiply large numbers by 9</b>.</p>	

	<p>Develop fluency in multiplication using efficient methods and recognising the properties of multiples. For example:</p> <ul style="list-style-type: none"> <li>• Multiply and divide by 10, 100 or 1000 fluently using place value.</li> </ul> <p>Knowledge of times tables to <b>multiply multiples</b></p> <p><b>Multiply 3 numbers</b> efficiently using <b>jottings</b> e.g. <math>4 \times 7 \times 9 =</math></p> <p>Apply knowledge of all times tables in <b>long multiplication</b>.</p>	<ul style="list-style-type: none"> <li>• Recognise large <b>multiples of 3 and 6</b> using the digital root.</li> </ul>
<p>Double and halving</p>	<ul style="list-style-type: none"> <li>• multiply by 5 by multiplying by 10 then halving, e.g. <math>18 \times 5 = 180 \div 2 = 90</math></li> <li>• multiply by 20 by doubling then multiplying by 10, e.g. <math>53 \times 20 = 106 \times 10 = 1060</math></li> </ul> <p>Multiply by 50 by multiplying by 100 and halving</p> <p>Multiply by 25 by multiplying by 100 and halving twice</p>	<p>Form equivalent calculations and use doubling and halving, e.g.</p> <ul style="list-style-type: none"> <li>• multiply by 4 by doubling twice, e.g. <math>16 \times 4 = 32 \times 2 = 64</math></li> <li>• multiply by 8 by doubling three times, e.g. <math>12 \times 8 = 24 \times 4 = 48 \times 2 = 96</math></li> <li>• divide by 4 by halving twice, e.g. <math>104 \div 4 = 52 \div 2 = 26</math></li> <li>• divide by 8 by halving three times, e.g. <math>104 \div 8 = 52 \div 4 = 26 \div 2 = 13</math></li> </ul>
<p>Written multiplication and division</p>	<p>multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers</p> <p>§ multiply and divide numbers mentally drawing upon known facts</p> <p>§ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context</p>	

	<p>§ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000</p> <p>recognise and use square numbers and cube numbers, and the notation for squared ( 2 ) and cubed ( 3 )</p>	
Fractions and decimals		<ul style="list-style-type: none"> <li>• add and subtract fractions with the same denominator and denominators that are multiples of the same number</li> <li>• multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams</li> <li>• solve problems involving number up to three decimal places</li> <li>• solve problems which require knowing percentage and decimal equivalents of 2 1 , 4 1 , 5 1 , 5 2 , 5 4 and those fractions with a denominator of a multiple of 10 or 25.</li> <li>• Find fractions of whole numbers or quantities, e.g. 2 3 of 27, 4 5 of 70kg Find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20kg, 10% of £80</li> </ul>
Mental strategies		<ul style="list-style-type: none"> <li>• count on or back in hundreds, tens, ones and tenths</li> <li>• partition: add hundreds, tens or ones separately, then recombine</li> <li>• subtract by counting up from the smaller to the larger number</li> <li>• add or subtract a multiple of 10 or 100 and adjust</li> <li>• partition: double and adjust</li> <li>• use knowledge of place value and related calculations, e.g. 6.3 – 4.8 using 63 – 48 <ul style="list-style-type: none"> <li>• partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)</li> </ul> </li> </ul>

#### Counting forwards and backwards

$3.2 + 0.6$	count on in tenths
$1.7 + 0.55$	count on in tenths and hundredths

#### Reordering



$12 + 17 + 8 + 3$	$12 + 8 + 17 + 3$
$25 + 36 + 75$	$25 + 75 + 36$
$58 + 47 - 38$	$58 - 38 + 47$
$200 + 567$	$567 + 200$
$1.7 + 2.8 + 0.3$	$1.7 + 0.3 + 2.8$
$3 + 8 + 7 + 6 + 2$	$3 + 7 + 8 + 2 + 6$
$34 + 27 + 46$	$34 + 46 + 27$
$180 + 650$	$650 + 180$ (thinking of 180 as 150 + 30)
$1.7 + 2.8 + 0.3$	$1.7 + 0.3 + 2.8$
$4.7 + 5.6 - 0.7$	$4.7 - 0.7 + 5.6 = 4 + 5.6$

Partitioning: counting on or back

$43 + 28 + 51$	$40 + 3 + 20 + 8 + 50 + 1 = 40 + 20 + 50 + 3 + 8 + 1$
$5.6 + 3.7$	$5.6 + 3 + 0.7 = 8.6 + 0.7$
$4.7 - 3.5$	$4.7 - 3 - 0.5$
$540 + 280$	$540 + 200 + 80$
$276 - 153$	$276 - 100 - 50 - 3$

Partitioning: bridging through multiples of 10

$607 - 288$	$288 + 12 + 300 + 7$
$6070 - 4987$	$4987 + 13 + 1000 + 70$
$1.4 + 1.7$ or $5.6 - 3.7$	$1.4 + 0.6 + 1.1$ or $5.6 - 0.6 - 3 - 0.1$
$0.8 + 0.35$	$0.8 + 0.2 + 0.15$
$8.3 - 2.8$	$2.8 + 0.2 + 5.3$ or $8.3 - 2.3 - 0.5$

Partitioning: compensating

$138 + 69$	$138 + 70 - 1$
$405 - 399$	$405 - 400 + 1$
$2\frac{1}{2} + 1\frac{3}{4}$	$2\frac{1}{2} + 2 - \frac{1}{4}$
$5.7 + 3.9$	$5.7 + 4.0 - 0.1$
$6.8 - 4.9$	$6.8 - 5.0 + 0.1$

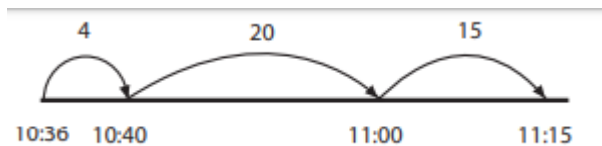
Partitioning: using 'near' doubles

$160 + 170$	is double 150, then add 10, then add 20 or double 160 and add 10 or double 170 and subtract 10
$2.5 + 2.6$	is double 2.5 and add 0.1 or double 2.6 and subtract 0.1

Partitioning: bridging through 60 to calculate a time interval

What time will it be 26 minutes after 3.30am?
What was the time 33 minutes before 2.15pm?
It is 4.18pm. How many minutes to 5.00pm? 5.26pm?

It is 08.35. How many minutes is it to 09.15?
It is 11.45. How many hours and minutes is it to 15.20?
A train leaves London for Leeds at 22.33. The journey takes 2 hours 47 minutes. What time does the train arrive?





## Year 6

Year 6	Autumn	Spring	Summer
Mental addition and subtraction	<ul style="list-style-type: none"><li>• addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place, e.g. <math>650 + \dots = 930</math>, <math>\dots - 1.4 = 2.5</math></li><li>• what must be added to a decimal with units, tenths and hundredths to make the next whole number, e.g. <math>7.26 + \dots = 8</math></li><li>• add or subtract pairs of decimals with units, tenths or hundredths, e.g. <math>0.7 + 3.38</math></li><li>• find doubles of decimals each with units and tenths, e.g. <math>1.6 + 1.6</math></li><li>• add near doubles of decimals, e.g. <math>2.5 + 2.6</math></li><li>• add or subtract a decimal with units and tenths, that is nearly a whole number, e.g. <math>4.3 + 2.9</math>, <math>6.5 - 3.8</math></li></ul>		

<p>Written addition and subtraction</p>	<p>add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)        § add and subtract numbers mentally with increasingly large numbers</p>	
<p>Multiplication and division</p>	<p>Recall <b>multiples of 12</b> in any order, including missing numbers and related division facts fluently.</p> <p>Recall <b>multiples of all times tables</b> up to 12x12 in any order, including missing numbers and related division facts with fluency</p> <ul style="list-style-type: none"> <li>• squares to <math>12 \times 12</math></li> </ul> <p>large <b>multiples of 3 and 6</b> using the digital root.</p> <p>knowledge of times tables to <b>multiply multiples</b></p> <p><b>Multiply 3 numbers</b> efficiently using <b>jottings</b> e.g. <math>4 \times 7 \times 9 =</math></p> <p>Apply knowledge of all times tables in <b>long multiplication</b>.</p> <ul style="list-style-type: none"> <li>• division facts corresponding to tables up to <math>10 \times 10</math>, and the related unit fractions, e.g. <math>7 \times 9 = 63</math> so one-ninth of 63 is 7 and one-seventh of 63 is 9</li> <li>• percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths</li> <li>• factor pairs to 100</li> <li>• Cube numbers</li> </ul> <p>Multiply by 10 and halve to multiply large numbers by 5.</p>	<p>Develop <b>fluency</b> in multiplication, using <b>efficient methods</b> and <b>recognising the properties of multiples</b>. For example:</p> <p><b>and divide by 10, 100 or 1000</b> fluently using place value.</p>

	<p>by 10 and subtract one group to <b>multiply large by 9</b>.</p> <ul style="list-style-type: none"> <li>• Double and double again to multiply large numbers by 4.</li> </ul>	
<p>Double and halving</p>	<p>Double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2 Form equivalent calculations and use doubling and halving, e.g. • divide by 25 by dividing by 100 then multiplying by 4 e.g. <math>460 \div 25 = 4.6 \times 4 = 18.4</math> • divide by 50 by dividing by 100 then doubling e.g. <math>270 \div 50 = 2.7 \times 2 = 5.4</math></p>	
<p>Written multiplication and division</p>	<p>multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication</p> <p>§ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</p> <p>§ divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context</p> <p>§ perform mental calculations, including with mixed operations and large numbers</p> <ul style="list-style-type: none"> <li>• division facts corresponding to tables up to <math>10 \times 10</math>, and the related unit fractions, e.g. <math>7 \times 9 = 63</math> so one-ninth of 63 is 7 and one-seventh of 63 is 9</li> </ul>	

	<p>Pupils explore the order of operations using brackets; for example, <math>2 + 1 \times 3 = 5</math> and <math>(2 + 1) \times 3 = 9</math>.</p>	
<p>Fractions</p>	<p>add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions</p> <p>multiply simple pairs of proper fractions, writing the answer in its simplest form</p> <p>divide proper fractions by whole numbers</p> <p>multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places</p> <p>multiply one-digit numbers with up to two decimal places by whole numbers &amp; use written division methods in cases where the answer has up to two decimal places</p> <p>solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison</p> <p>Recall equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or <math>\frac{35}{100}</math> Find half of decimals with units and tenths, e.g. half of 3.2 Find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g</p> <ul style="list-style-type: none"> <li>• Counting in tenths and decimals on counting stick</li> </ul>	

## Mental Strategies

- count on or back in hundreds, tens, ones, tenths and hundredths
- use knowledge of place value and related calculations, e.g.  $680 + 430$ ,  $6.8 + 4.3$ ,  $0.68 + 0.43$  can all be worked out using the related calculation  $68 + 43$
- use knowledge of place value and of doubles of two-digit whole numbers
- partition: double and adjust
- partition: add or subtract a whole number and adjust, e.g.  $4.3 + 2.9 = 4.3 + 3 - 0.1$ ,  $6.5 - 3.8 = 6.5 - 4 + 0.2$
- partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times, 12-hour and 24-hour clock)

Counting forwards and backwards

$3.2 + 0.6$	count on in tenths
$1.7 + 0.55$	count on in tenths and hundredths

Reordering

$12 + 17 + 8 + 3$	$12 + 8 + 17 + 3$
$25 + 36 + 75$	$25 + 75 + 36$
$58 + 47 - 38$	$58 - 38 + 47$
$200 + 567$	$567 + 200$
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$276 - 153$	$276 - 100 - 50 - 3$

Partitioning: bridging through multiples of 10

$607 - 288$	$288 + 12 + 300 + 7$
$6070 - 4987$	$4987 + 13 + 1000 + 70$
$1.4 + 1.7$ or $5.6 - 3.7$	$1.4 + 0.6 + 1.1$ or $5.6 - 0.6 - 3 - 0.1$
$0.8 + 0.35$	$0.8 + 0.2 + 0.15$
$8.3 - 2.8$	$2.8 + 0.2 + 5.3$ or $8.3 - 2.3 - 0.5$

Partitioning: compensating

$138 + 69$	$138 + 70 - 1$
$405 - 399$	$405 - 400 + 1$
$2\frac{1}{2} + 1\frac{3}{4}$	$2\frac{1}{2} + 2 - \frac{1}{4}$
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$160 + 170$	is double 150, then add 10, then add 20 or double 160 and add 10 or double 170 and subtract 10
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Partitioning: bridging through 60 to calculate a time interval

What time will it be 26 minutes after 3.30am?

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It is 11.45. How many hours and minutes is it to 15.20?

A train leaves London for Leeds at 22.33.  
The journey takes 2 hours 47 minutes.  
What time does the train arrive?

